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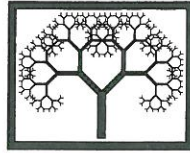
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# Efficient Finite Element Calculation of $N_\gamma$

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**Keywords:** Mohr-Coulomb, bearing capacity, implicit integration, stress update, elasto-plastic constitutive matrix, non-linear FEM.

This paper deals with the computational aspects of the Mohr-Coulomb material model, in particular the calculation of the bearing capacity factor  $N_\gamma$  for a strip and circular footing. The main focus is on the stress update algorithm for the elasto-plastic finite element computations. The merits of a return mapping algorithm which carries out all the manipulations in principal stress space will be examined and exemplified for a Mohr-Coulomb material. The method was first presented in ref. [1].

By simple geometric means methods of determining which type of stress return should be invoked are presented. For each type of return, i.e. return to a plane, return to a corner and return to the apex, explicit formulae for the updated stress and the corresponding constitutive matrix is presented. When expressed in the principal stress space these formulae turn out to be extremely simple, when compared to the return mapping implementation in the general stress space of for example Crisfield [2].

The Mohr-Coulomb material model is known to cause problems in numerical calculations, due to the singularities present at the corners and at the apex. In some applications these problems are overcome by slightly modifying the criterion by rounding of the corners and thereby eliminate the singularities. The drawback of this modification is that calculated results, in general, will not converge towards the exact solutions. With the presented method the singularities are handled in a simple and robust manner and therefore no problems are experienced, and it is shown that the obtained results converge toward the exact values with great precision. Also the efficiency of different methods of handling the corner and apex singularities is evaluated.

Some comments are also given as how to mend some of the problems of the general return mapping implementation of Crisfield [2].

## References

- [1] J. Clausen, L. Damkilde and L. Andersen. Efficient return algorithms for associated plasticity with multiple yield planes, *International Journal for Numerical Methods in Engineering*, 66:1036–1059, 2006.
- [2] M.A. Crisfield. *Non-Linear Finite Element Analysis of Solids and Structures. Vol. 2: Advanced topics*, John Wiley & Sons, 1997.